Probability

3.3 Fundamental Counting Rule and Combinations

1. A simple puzzle called a Jumble involves unscrambling the letters of a word. An example of this kind of puzzle is given here: HALWET. How many unique arrangements of these letters are possible? What is the scrambled word?

2. The iphone has the option to set up a four digit pin to protect the phone. How many unique pins can be created using four digits if the user decides to not repeat any digit? What's the probability that someone who finds a phone in the university library randomly picks the correct four digit pin to open it assuming the owner used this sort of pin involving 4 unique digits?

3. A gym locker is protected by the Master Lock combination lock pictured below. This type of lock requires you to enter a 4-digit combination. Assuming the digits can be any value from 0 to 9, how many different combinations are possible? Would it be likely to randomly enter a 4-digit combination at someone else's locker and have it open?

4. I have an idea for a simple lottery which would involve picking a random arrangement of the letters: A B C D E F G H I J. How many unique arrangements are possible? Could this be a successful lottery strategy if I charge $1 per ticket (per arrangement), and I pay the winner $1.5 million for picking the correct arrangement?

5. A local ice cream shop has 27 flavors to choose from. If you must pick three different flavors (without repeating any flavor) to form your ice cream sundae, how many different ice cream sundaes are possible?

6. If you had a playlist in your iTunes that contains 30 songs you enjoy listening to when exercising and you would like to pick 6 songs from that set to listen to during a two-mile run, how many sets of 6 songs can you create from the set of 30 songs in your playlist (assume that the order of the 6 songs does not matter since you will listen to the songs on shuffle)? How many sets of 6 songs arrangements would be possible if you also treated the order in which the 6 songs played as being important (in other words, the same 6 songs played in a different order represent a different set)?

7. A sorority on campus has 20 honors students as members. If the sorority wants to choose 4 girls to represent the sorority in Orlando for a Brain Bowl competition, how many different groups of four can be drawn from the set of 20 honors students in their sorority?

Answers:

1. $6*5*4*3*2*1=6!=720$ The word is WEALTH.

2. Since there are ten digits (0-9) to choose from for each spot, there are $10*9*8*7=5,040$ different pins possible. The probability we would guess the specific pin is $\frac{1}{5,040}$.

3. Since there are ten digits (0-9) to choose from for each spot, there are $10*10*10*10=10,000$ different combinations possible. The probability we would guess the specific combination is $\frac{1}{10,000}$, so it would be unusual to guess the combination.

4. There are $10!=3,628,800$ arrangements possible because we have ten choices for the first position, nine letters left to choose for the second position, … , one left for the last position. If it costs a dollar to buy each arrangement, then it would cost $3,628,800 to buy all of the combinations. I would only need to pay out $1.5 million to the winner. Financially, it would be a good lottery; however, I would have to make sure I didn't sell the tickets to the whole state of Florida since the population is so much larger than the number of unique arrangements available. If I sold tickets only in Miami, it would be a good game.

5. ${}_{27}C_3=2,925$ different sundaes possible.

6. There are ${}_{30}C_6 = 593,775$ sets of 6 songs possible if we ignore the order. If we do not ignore the order of the 6 songs, $30*29*28*27*26*25 = {}_{30}P_6 = 427,518,000$ are possible.

7. There are ${}_{20}C_4 = 4,845$ different groups possible.